

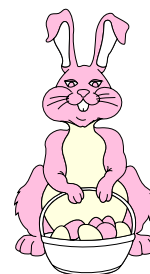
The Mathematics of Easter

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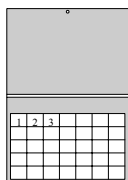
Mathematics appears in the most unexpected places. Note for example the curious concerns of Sunday afternoon strollers as to whether or not the seven bridges of Königsberg could be traversed in a single trip without crossing any bridge more than once. Note also the consequential Königsberg Bridge Problem as pursued by Euler and the early traces of network theory. And then there is the casual coloring of a map of England and the ultimate formulating of the Four-Color Map Conjecture, a highly challenging topological problem resolved only in the late twentieth century.

The calendar likewise shares in this phenomenon of the emergence of enormously difficult mathematical questions but in an unexpected manner. According to the Council of Nicaea in A.D. 325, the Easter date was fixed as the first Sunday after the first full moon on or after the vernal equinox. Such a rule contained within it all the potential for a mathematical problem of considerable dimension, all the more so as the projection of Easter dates into the distant future is considered. Critical to the analysis of this problem is a three-fold consideration of cycles, namely, those of a solar, a lunar, and a weekly kind. Students will enjoy noting the mathematical subtleties of the calendar, actual use of the Easter date formula, and exploring the extended questions that such an activity implies.



The Gregorian Calendar

The Gregorian calendar, now in worldwide civil use, dates from 1582 when the ancient Julian Calendar underwent a major modification. Motivated by seasonal concerns, October 4, Thursday, 1582 was followed by October 15, Friday, 1582. Moreover, centesimal years were not to be counted as leap years unless divisible by 400. Hence, 1900 would not be a leap year but 2000 was. Some today, for ecclesiastical reasons, celebrate Easter in accordance with the ancient Julian calendar. Interestingly, both Western and Eastern computations identify Easter as April 15 this year (2001).



Under the heading of the Gregorian calendar, Easter can thus occur as early as March 22 (last happening in 1818 and next occurring in 2285) and as late as April 25 (last happening in 1943 and next occurring in 2038). There are 35 possible Easter dates.

Although the Gregorian calendar has a period of date-day repetition equal to 400 years, the Easter period is 5,700,000 years (see reference 1). Accordingly, the least frequent Easter date became determinate and proves to be March 22. The most common is April 19. Other results stem from such an analysis and include, for example, the fact that Easter cannot occur in March two years in a row. These observations are based on a year-by-year consideration of actual Easter dates (see reference 1).

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Variation on the Gaussian Easter Formula

The sequence of steps that permits calculating the date of Easter for a particular year is given below. Computations of necessity take into account solar, lunar, and weekly calendar patterns, patterns that are immensely difficult to justify in a brief descriptive account.

1. $\frac{\text{year}}{19} = A \text{ plus remainder } B$
2. $\frac{\text{year}}{100} = C \text{ plus remainder } D$
3. $\frac{C}{4} = E \text{ plus remainder } F$
4. $\frac{C+8}{25} = G \text{ (discard remainder)}$
5. $\frac{C+1-G}{3} = H \text{ (discard remainder)}$
6. $\frac{19B+C+15-(E+H)}{30} = \text{quotient (discard) plus remainder } Z$
7. $\frac{D}{4} = K \text{ plus remainder } L$
8. $\frac{2F+2K+32-(Z+L)}{7} = \text{quotient (discard) plus remainder } N$
9. $\frac{B+11Z+22N}{451} = P \text{ (discard remainder)}$
10. $\frac{Z+N+114-7P}{31} = Q \text{ plus remainder } R$

$$\frac{Z + N + 114 - 7P}{31} = Q$$

Then Q denotes the month and $R + 1$ denotes the day on which Easter falls for a given year. An illustration reinforces the formula. To calculate Easter for the year 2001, the following letter values are obtained:

1. $A = 105, B = 6$
2. $C = 20, D = 1$
3. $E = 5, F = 0$
4. $G = 1$
5. $H = 6$
6. $Z = 18$
7. $K = 0, L = 1$
8. $N = 6$
9. $P = 0$
10. $Q = 4, R = 14$

As Easter is given by month Q and day $R + 1$, the actual date of Easter for 2001 is April 15. This same day is a common Easter date of the twenty-first century and occurs again in the years 2063, 2074, 2085, and 2096. It proves to be a fairly common Easter date overall and occurs exactly 192,850 times in the 5,700,000 year Easter cycle. Interestingly, both Eastern and Western worlds will commemorate Easter on the same date, April 15, in 2001. The reader may wish to apply the above formula to the year 2002, a year in which Easter occurs on March 31.

The Easter Sunday Frequency table below permits a quick glance at the complete pattern for the twenty-first century.

TABLE I
EASTER SUNDAY FREQUENCY
the twenty-first century (2001 - 2100)

March 22	0
March 23	1
March 24	0
March 25	2
March 26	3
March 27	2
March 28	3
March 29	3
March 30	4
March 31	5
April 1	3
April 2	2
April 3	3
April 4	4
April 5	4
April 6	3
April 7	2
April 8	3
April 9	3
April 10	4
April 11	4
April 12	4
April 13	2
April 14	3
April 15	5
April 16	4
April 17	3
April 18	3
April 19	3
April 20	5
April 21	4
April 22	2
April 23	1
April 24	2
April 25	1

With computer assistance, some presently unanswered questions come within reach. Notable among them is the determination of the shortest interval of time in which all possible Easter dates occur. This time frame may be the interval extending from 1799 to 1886, a span of eighty-seven years (see reference 2).

All in all, the Easter Date Problem easily generates many other challenging numerical pursuits. The tools of number theory and computer analysis lend themselves in large measure to the solution of such problems.

References

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5. T. H. O'Bierne, *Puzzles and Paradoxes*, Oxford University Press, New York, 1965.